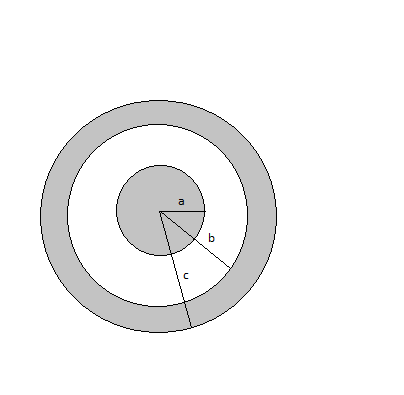
Gauss’s Law Problems

**Question 2**. An insulating sphere with charge q uniformly distributed throughout its volume is surrounded by a metal shell with charge 2q. Write down an expression for the electric field in the region r < a, a < r < b, b < r < c, and c < r.





**Extra Credit (5 pts.)**

An early model for the nucleus of an atom was a solid sphere of radius R with charge Q evenly distributed throughout the volume. Use Gauss’s law to determine an expression for the electric field strength inside the sphere at radius r < R, in terms of ε0, Q, and R.

From Gauss’s law we have:

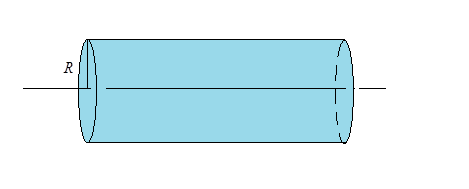


**Problem**

Do dust problem. Say have charge carries q = 1pC. And about 1 particle per cm3 (same density as air about). What radius of cloud would result in E = 12.6kV/m – dielectric strength of air?

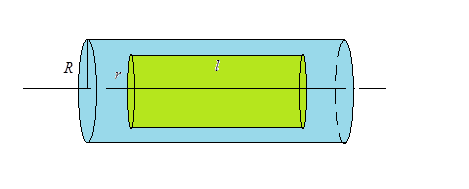
**Problem**

Suppose you have an infinitely long cylinder with radius R, and radial charge density ρ(r) = r2 Coulomb/m3. What is the field at a distance r < R, and r > R from the center of the cylinder?



**Solution**

We use Gauss’s law to derive the field. So draw a Gaussian cylinder going through the radius r and of length ℓ.



Then Gauss’s law says:



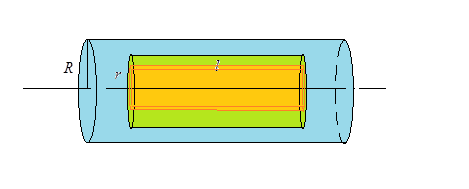
Now the field **E** = E**r**, and area vector d**A** on the Gaussian cylinder are pointed radially outward, away from the central axis. So we can say:



We can pull the field outside the integral because it is constant over the surface of the cylinder, and then we’re left with ∫dA = 2πrℓ.



So far this is completely general. But how do we get Qenclosed? We write it as Qenclosed = ∫ρdV. To get dV we can break up the Gaussian cylinder into cylindrical shells of radius r (different r than above; this is radius of orange shell – I’m running out of different names for these radii) and thickness dr. Such a shell would have volume dV = surface area × thickness = 2πrℓdr.



So then we have:



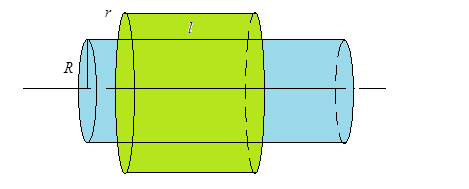
Filling this in we get:



and so,



For r > R, we use Gauss’s law again.



The only thing different is Qenclosed, which is:



(upper bound on integral only goes to R since no more charge outside that radius) Filling this in,



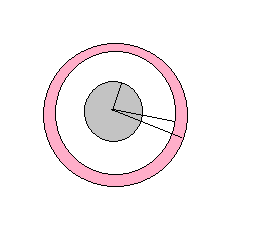
and so we have:



They do match, as they should, when r = R.

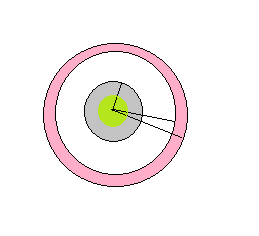
**Problem**

An insulating sphere of radius R1 has charge Q1 uniformly distributed throughout its interior. It is surrounded by a metallic shell of radius R2, and R3 and charge Q2. What is the field at r < R1, R1 < r < R2, R2 < r < R3, and r > R3?



**Solution**

We draw a Guassian surface of radius r < R1 to get the field inside the insulator.



Then according to Gauss’s law:



Now the field will be pointing radially outward, same as d**A**. So we can write:



and what is Qenclosed? Qenclosed = ∫ρdV. For spherical systems we can write dV as the volume of concentric sphere, similar to the concentric cylinders previously. Concentric spheres would have a volume dV = surface area × thickness = 4πr2dr. So then,



Note that ρ is constant, equal to Q1/(Volume of insulating sphere) since the charge was said to be uniformly distributed. And so ρ = Q1/(4πR13/3). Filling this into Qenclosed we get:



and so the field is:



so we have:



For r in between R1 and R2 doing the Gaussian surface thing again we have:



and so,



For R2 < r < R3 we have:



since we’re inside a metal. And for r > R3 we have:

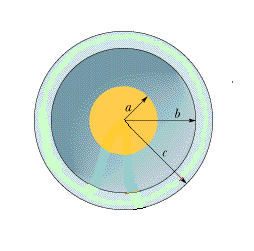


and so,



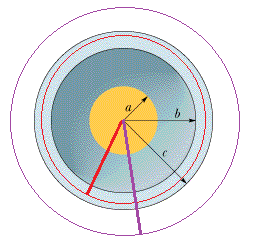
**Problem**

In the figure a solid sphere of radius *a* = 2cm is concentric with a spherical conducting shell of inner radius *b* = 4cm, and outer radius *c* = 5cm. The sphere has a net uniform charge *q*1 = 7fC; the shell has a net charge *q*2 = -3fC. What is the magnitude (in N/C) of the electric field at radial distances (a) *r* = 4.5cm, (b) r = 6cm? And finally, what is the charge on the inner surface of the spherical shell?



**Solution**

Figure looks like this:



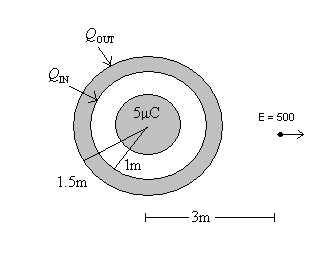
(a) The radius r = 4.5cm is inside the metallic shell so E = 0 there.

(b) The purple circle is a Gaussian sphere with radius 6cm. The electric field inside the sphere is given by E = kQenclosed/r2 = k(q1 + q2)/r2 = (9×109)(7×10-15­­­ – 3×10-15)/(0.06)2 = 0.01 N/C.

(c) The charge on the inner surface of the spherical shell is equal/opposite to the charge on the inner sphere = -7×10-15C.

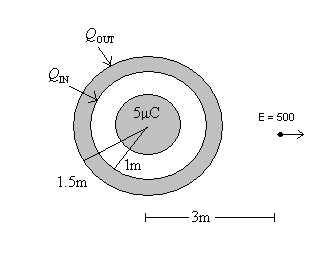
**Problem**

Consider the arrangement below. We have a central metallic sphere with net charge 5μC, and surrounding it, a metallic shell with an unknown net charge Q, with inner radius 1m, and outer radius 1.5m. Now suppose that the electric field strength at a distance of 3m is E = 500N/C and pointing outward. Determine the net charge on the inner (QIN) and outer surfaces (QOUT) of the metallic shell (in μC).



**Solution**

Looks like this:

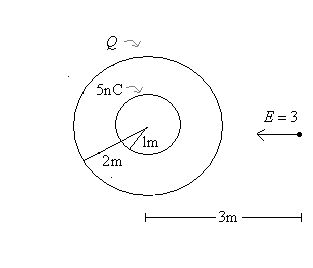


The net charge on the inner surface is QIN = -5μC since it must cancel out the 5μC charge in order to make the **E** field zero inside the outer conductor. Using Gauss’s law again we can determine the outer charge QOUT. Drawing a Gaussian sphere through the point r = 3m, we get:



And the enclosed charge is Qenc. = 5μC + Q­IN + QOUT = 5μC – 5μC + QOUT, which implies that QOUT = 0.5μC.

10. Suppose you have a two concentric spheres of radii 1m and 2m respectively. If the charge on the smaller one is 5nC, and the electric field at a distance of 3m is **E** = 3N/C pointing inward, what is the charge on the 2m sphere?





Applying to the 3m point,



This requires therefore that Q + 5nC = -3nC → Q = -8nC.